Int. J. Heat Mass Transfer. Vol. 31, No. 7, pp. 1477–1482, 1988
Printed in Great Britain

0017-9310/88 $3.00+0.00

Evaluation of flux models for radiative transfer in rectangular furnaces

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(*Received* 23 *March* 1987)

Abstract—Three flux-type models for three-dimensional radiative heat transfer were applied to the prediction of the radiative flux density and the source term of a box-shaped enclosure problem based on data reported previously on a large-scale experimental furnace with steep temperature gradients. The models, which are a six-term discrete ordinate model and two Schuster–Schwarzschild type six-flux models, were evaluated from the viewpoints of both predictive accuracy and computational economy by comparing their predictions with exact solutions produced previously. The comparison showed that the six-flux model based on angular subdivisions related to the enclosure geometry produces more accurate results and is computationally less expensive than the other two models.

1. INTRODUCTION

THE MOST accurate procedures available for mathematical modelling of radiation fields within furnaces are the zone [1, 2] and Monte-Carlo [3, 4] methods, both of which have been extensively and successfully applied to the prediction of radiant heat transfer in furnaces for which complete knowledge of the flow and concentration fields was available. However, these radiation models have not been extensively used as part of the complete prediction procedure. The reason for this is that the equations modelling the radiation field are not differential in form and hence are not well suited to solution simultaneously with the differential equations for flow and reaction. In order to overcome this disadvantage, flux models [5–9] have been widely employed as alternative, but less accurate, procedures in complete prediction procedures. Flux models of radiation fields take the form of partial differential equations which are amenable to solution simultaneously and conveniently with the equations for flow and chemical reaction.

Previously published multidimensional evaluations of the accuracy of flux models of radiation fields have taken two forms.

(1) The flux model has been employed as part of a complete prediction procedure and predicted temperature and radiative heat flux distributions have been compared with experimentally determined data [5, 10, 11]. With this procedure, it is impossible to decide whether discrepancies between the predictions and measurements are attributable directly to the flux model employed or to inaccuracies in the submodels used for the prediction of flow, reaction, etc.

(2) The flux model has been tested in isolation from the modelling of other physical processes by using a prescribed uniform radiative energy source term distribution and comparing predicted temperature and radiative heat flux distributions with values predicted using the zone or Monte-Carlo methods [7, 9, 12]. This procedure for the evaluation of the accuracy of a flux model suffers from two major disadvantages: (a) even if acceptably accurate predictions are obtained for the uniform source term distributions, there is no certainty that similarly accurate predictions will be produced for the highly non-uniform distributions encountered in operating furnaces and combustors; (b) considering the iterative sequence of solution in complete prediction procedures, it is obvious that when testing a radiation model which is intended for use in a complete prediction procedure, the input data provided should be complete temperature distributions, and the predicted and tested quantities should be the radiative flux density and radiative energy source term distributions.

What is required at the present time is the evaluation of the predictions of these flux-type radiation models in isolation from the models of flow and reaction and under the conditions typically encountered in industrial furnaces.

The first radiation model is a six-term discrete ordinate model for a three-dimensional radiation field derived in ref. [13]. In this method, the detailed angular distribution of radiation intensity is approximated by a finite number of intensities in discrete directions spanning the solid angle at each point. Application of the equation of radiant energy transfer into each direction produces partial differential equations in terms of the unknown intensities in the specified directions. Any angular integral of intensity at a point may be found from the discrete intensities by using a numerical quadrature formula.

The second and third radiation models are: (a) a Schuster–Schwarzschild type six-flux model based on six equal subdivisions of the solid angle surrounding a point and (b) a Schuster–Schwarzschild type six-

NOMENCLATURE

q component of radiative flux density vector $[W\ m^{-2}]$
Q source term for radiative energy $[W\ m^{-3}]$
x, y, z rectangular Cartesian coordinates [m].

Superscript
$\tilde{}$ dimensionless.

flux model with angular subdivisions related to the enclosure geometry. Both models have previously been derived in ref. [7]. The basis of these models is to subdivide the total solid angle surrounding a point into six pyramid-shaped smaller solid angles in each of which intensity is assumed to be uniform. The smaller solid angles are taken to be those subtended by the six faces of the rectangular enclosure at the point under consideration. Discontinuous changes in intensity, therefore, occur in passing from one smaller solid angle to any adjacent smaller solid angle. Integration of the equation of radiant energy transfer for each smaller solid angle, in turn, produces six first-order differential equations in the unknown intensities.

In this paper, these radiation models are applied to the prediction of distributions of radiative flux density and the radiative energy source term of a rectangular enclosure problem. The problem is based on data taken from a large-scale experimental furnace with steep temperature gradients. The three flux-type radiation models are tested from the points of view of both accuracy and computational economy by comparing their predictions with exact values reported previously in ref. [14].

2. THE TEST PROBLEM

The flux-type models considered have been tested by making predictions for a black-walled enclosure problem for which exact solutions have been produced previously [14]. The enclosure problem is based on data reported by Strömberg [15] on a large-scale experimental furnace with steep temperature gradients typically encountered in industrial furnaces.

The experimental furnace under consideration is horizontal, of tunnel type with a square cross-section. It is fired horizontally from the centre of the left end wall, which is the burner wall, with a mixture of oil and gas with no swirl, and operates at atmospheric pressure. The four side walls are water cooled, and the burner and back end walls are refractory. A detailed description of the data obtained from the experimental furnace and used as input data for flux-type models can be found elsewhere [14].

3. NUMERICAL SOLUTION PROCEDURE

The partial differential equations representing the radiation models under consideration have been recast into finite difference forms by using the control volume approach. As the variation of gas and wall temperatures about the z-axis is symmetrical, and identical in both x- and y-directions, it is only necessary to calculate the values of the components of the radiative flux density vector and radiative energy source term for one quarter of the cross-section. One quarter of the enclosure has been subdivided into $2 \times 2 \times 24$ control volumes in the x-, y- and z-directions, respectively. A medium grid point lies at the geometrical centre of each control volume and a surface grid point lies at the centre of each control volume face in contact with the walls of the enclosure. Hence the total number of medium and surface grid points are $3 \times 2 \times 2 \times 24$ and $2(2 \times 2 + 2 \times 24 + 24 \times 2)$, respectively. The resulting sets of simultaneous algebraic equations were solved by the iterative procedure developed by Peaceman and Rachford [16] for numerical solution of the algebraic equations with a coefficient matrix of the tri-diagonal type. This procedure can be described as 'forward elimination followed by backward substitution'.

4. EVALUATION OF THE FLUX MODEL PREDICTIONS

Point values of the dimensionless radiative energy source term and flux density for $2 \times 2 \times 24$ medium grid points in one quarter of the test enclosure have been produced using:

(a) the six-term discrete ordinate model—Model 1;
(b) the six-flux model, utilizing six equal subdivisions of the total solid angle surrounding any point within the enclosure—Model 2;
(c) the six-flux model, utilizing subdivisions of the total solid angle based upon the geometry of the enclosure under consideration—Model 3.

The predictions of these models have been compared with the exact solutions reported previously in the literature [14].

In the discussion that follows, all physical quantities are expressed in dimensionless forms which are obtained by dividing them by the shortest dimension of the enclosure or by the maximum emissive power of the gas, depending on the quantity.

4.1. *Source term distributions*

Figure 1 shows comparison between flux model predictions of dimensionless source term distributions and

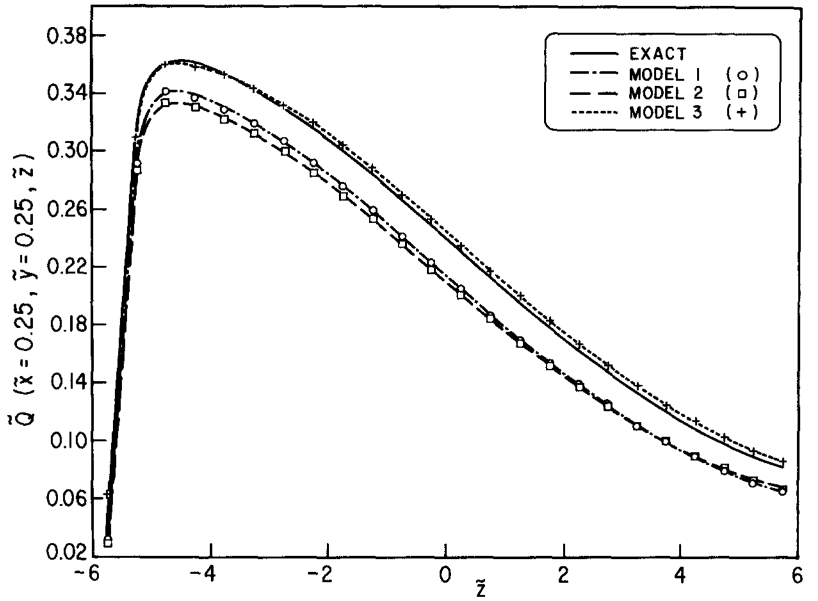

FIG. 1. Comparison between the exact values and flux model predictions of dimensionless radiative energy source terms along ($\tilde{x} = 0.25$, $\tilde{y} = 0.25$, $\tilde{z}$).

the exact values for points ($\tilde{x} = 0.25$, $\tilde{y} = 0.25$, $\tilde{z}$). These grid points represent the points at the centre of the row of control volumes nearest to the furnace axis. It can be seen that the exact source term distribution follows the physically expected trend, rising steeply from the burner wall onwards, going through a maximum and decreasing continuously towards the exit. The maximum of the source term distribution occurs at the same location as the maximum of the temperature distribution. It can also be noted that the trend of the distributions predicted by the flux models is the same as that of the exact distribution and that Model 3 produces a better agreement with the exact solution.

Figure 2 illustrates the comparison between the exact values of dimensionless source term and the distributions predicted by the flux models for grid points ($\tilde{x} = 0.75$, $\tilde{y} = 0.25$, $\tilde{z}$) and ($\tilde{x} = 0.75$, $\tilde{y} = 0.75$, $\tilde{z}$). These grid points represent the medium points nearer to the side wall and near the corner of the furnace, respectively. It can be seen that good agreement is obtained and that the source term distributions for grid points ($\tilde{x} = 0.75$, $\tilde{y} = 0.75$, $\tilde{z}$) show smaller variation along the length of the furnace than those for other medium grid points. This is consistent with the uniform temperature distribution in the medium near the corner of the enclosure.

A condensed comparison of the flux model predictions of the dimensionless source term values is contained in Table 1. Three values are given for each

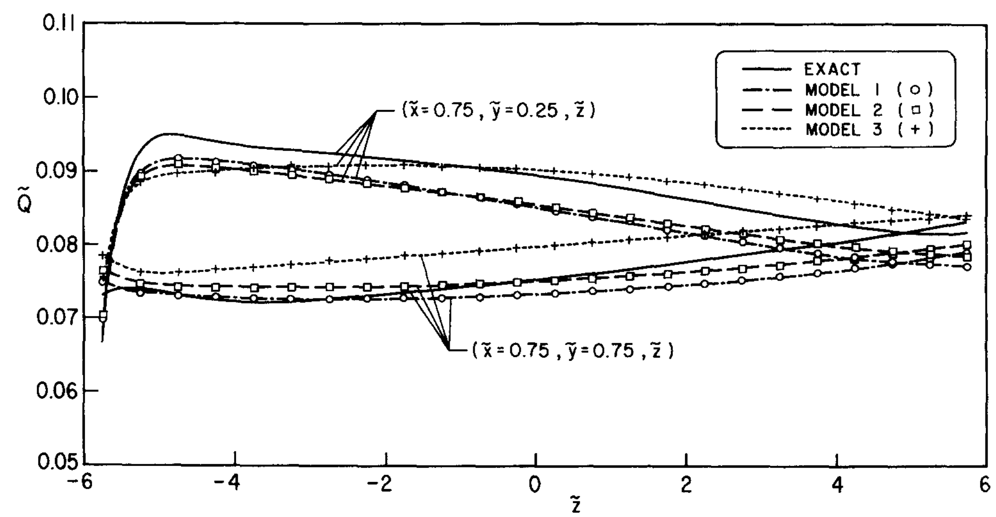

FIG. 2. Comparison between the exact values and flux model predictions of dimensionless radiative energy source terms along ($\tilde{x} = 0.75$, $\tilde{y} = 0.25$, $\tilde{z}$) and ($\tilde{x} = 0.75$, $\tilde{y} = 0.75$, $\tilde{z}$).

model; the maximum point percentage error and the average absolute percentage error both of which give measures of the accuracy of the predicted source terms, the number of iterations necessary to produce convergence of the iterated solution to within 0.0001% of the values in the previous iteration at all grid points, which measures the computing time.

As can be seen from Table 1, Model 3 produces more accurate results and is computationally less expensive than the other two models.

4.2. *Flux density distributions*

Figure 3 illustrates the comparison between the point values of the dimensionless flux density to the side wall in the positive x-direction predicted by the flux models and exact solutions for surface grid points. Points on the lines ($\tilde{x} = 1$, $\tilde{y} = 0.25$, $\tilde{z}$) and ($\tilde{x} = 1$, $\tilde{y} = 0.75$, $\tilde{z}$) represent points near the centre of the face and near the corner of the face, respectively. It can be seen that the flux densities to the wall are underestimated over the whole length of the enclosure by Models 1 and 2, and overestimated by Model 3.

A condensed comparison of the flux model predictions of the dimensionless flux densities is contained in Table 2. As can be seen from Table 2, the average absolute error produced by Model 3 is slightly higher than that predicted by Model 1. However, the maximum point percentage error produced by Model 3 is significantly lower than those predicted by the other two models. It can be noted that Model 3 produces more accurate results and is computationally less expensive.

In an earlier paper [7], these radiation models had been applied to a cubic enclosure problem with a uniform radiative energy source term. When the maximum point percentage and the average absolute errors in flux densities produced by the cubic enclosure problem are compared with the values found in this study, the errors calculated in this study are found to be approximately two times those produced in the previous study. This can be, to a large extent, due to the use of a highly non-uniform temperature distribution, and, to some extent, due to the use of an exact solution, as opposed to the Monte-Carlo solution, for testing purposes.

In order to test the effect of degree of subdivision

Table 1. Comparison of flux model predictions of dimensionless source terms

Flux model	Maximum percentage error	Average absolute percentage error	Number of iterations
Model 1	34.84	6.58	10
Model 2	40.23	6.67	13
Model 3	−26.61	3.65	9

Table 2. Comparison of flux model predictions of dimensionless flux densities to the side wall

Flux model	Maximum percentage error	Average absolute percentage error	Number of iterations
Model 1	51.80	14.92	10
Model 2	47.71	18.68	13
Model 3	39.50	16.31	9

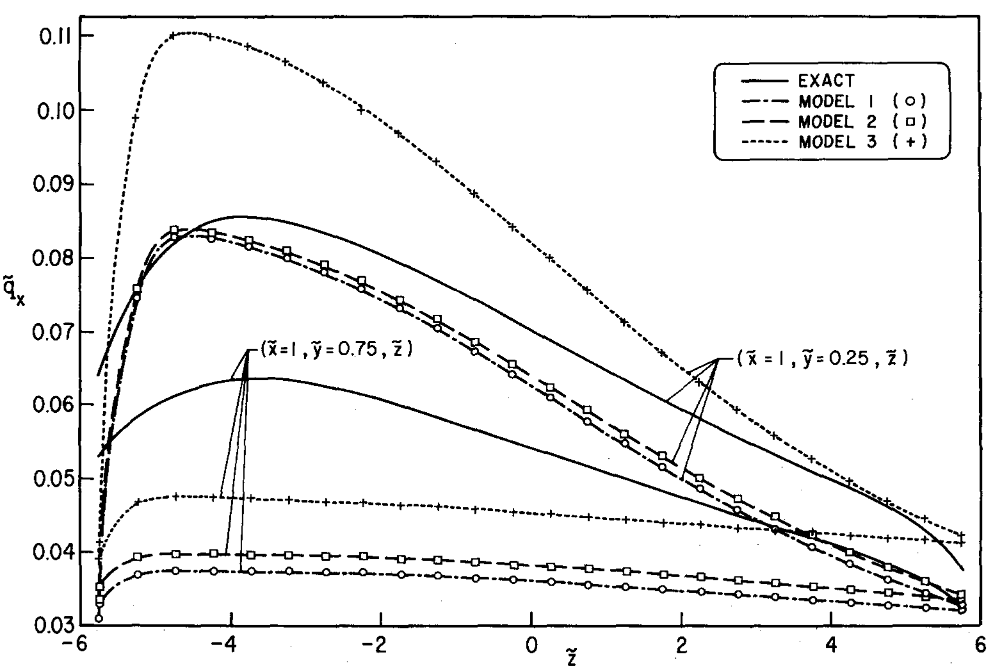

FIG. 3. Comparison between the exact values and flux model predictions of dimensionless flux densities to the side wall.

Table 3. Comparison of flux model predictions of the percentage errors in generated and removed radiative energy

Flux model	Percentage error in generation	Percentage error in removal
Model 1	1.32	1.32
Model 2	1.32	1.32
Model 3	1.44	1.44

on the accuracy of flux model predictions of point values of the dimensionless radiative energy source term and the flux density, the flux model programme has been run with a finer grid, $6 \times 6 \times 24$ control volumes. It has been found that the finer subdivision does not produce any substantial improvement on the predicted results, and that the slight changes in the point values can only be obtained at the expense of substantially increased computing time and storage.

To provide a global check on the accuracy of the flux model predictions, the total rate of removal of radiative energy through the walls and the total rate of generation of radiative energy within the enclosed medium were calculated and compared with the exact values. Table 3 shows the errors in generated and removed radiative energy produced by the flux model predictions. It can be seen that the percentage errors in generated and removed radiative energy are almost equal for each model, implying that each model produces consistent results, although different from the exact values.

5. CONCLUSION

Three flux-type models for box-shaped enclosures filled with an absorbing–emitting medium of constant properties have been applied to the prediction of the distributions of radiative flux density and the energy source term of a black-walled enclosure problem. The problem is based on data reported previously on a large-scale experimental furnace with steep temperature gradients typically encountered in industrial furnaces. The flux-type models which have been employed are a six-term discrete ordinate model and two Schuster–Schwarzschild type six-flux models; one utilizing six equal subdivisions of the total solid angle surrounding any point within the enclosure, and the other utilizing subdivisions of the total solid angle based upon the geometry of the enclosure. The models have been tested from the viewpoints of both predicted accuracy and computational economy by comparing their predictions with exact solutions reported earlier in the literature. The comparisons show that the six-flux model based on angular subdivisions related to the enclosure geometry produces more accurate results and is computationally less expensive than the other two models.

Results of the previous testings of the accuracy of the same models on a cubic enclosure problem with a uniform radiative energy source term and this study illustrate that the errors produced in the present investigation are approximately two times those calculated in the previous study. This implies that the evaluation of the accuracy of the radiation models under uniform radiative energy source term conditions can be misleading. The testing of any flux type model should be carried out on problems with a highly non-uniform gas temperature distribution typical of an operating furnace.

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EVALUATION DE MODELES POUR LE CALCUL DES FLUX RADIATIFS DANS DES FOURS RECTANGULAIRES

Résumé—Trois modèles pour l'estimation des transferts radiatifs tridimensionnels sont appliqués à un problème traité expérimentalement et dont les résultats ont déjà été publiés. Un modèle à six termes et deux modèles à six flux du type Schuster–Schwarzschild sont évalués du point de vue de la précision des prévisions et aussi de l'économie de calcul, en les comparant aux solutions précédemment produites. Le modèle à six flux, basé sur des subdivisions angulaires liées à la géométrie de l'enceinte, donne des résultats plus précis et il est comparativement moins cher que les deux autres modèles.

MODELLE ZUR BERECHNUNG DES STRAHLUNGS-WÄRMEAUSTAUSCHES IN QUADERFÖRMIGEN ÖFEN

Zusammenfassung—Es werden drei verschiedene Modelle für den dreidimensionalen Strahlungswärmeaustausch dazu verwendet, die Strahlungsdichte und den Quellterm in einem quaderförmigen Hohlraum zu berechnen. Die Grundlage hierfür bilden kürzlich veröffentlichte Meßdaten von einem Versuchsofen mit großen Temperaturgradienten. Die Modelle, ein sechsgliedriges Ordinaten-Modell und zwei Sechs-Fluß-Modelle nach Schuster–Schwarzschild werden nach Gesichtspunkten von Rechenzeit und Rechengenauigkeit durch Vergleich mit exakten Lösungen ausgewertet. Der Vergleich zeigt, daß das auf winkligen Einteilungen (entsprechend der Hohlraum-Geometrie) basierende Sechs-Fluß-Modell weniger kostenintensiv ist und zu genaueren Ergebnissen führt als die beiden anderen Modelle.

ОЦЕНКА МОДЕЛЕЙ ПОТОКА ПРИ РАДИАЦИОННОМ ПЕРЕНОСЕ В ПРЯМОУГОЛЬНЫХ ПЕЧАХ

Аннотация—Рассмотрены три модели трехмерного радиационного теплообмена, по которым но основе известных данных для полномасштабной печи с плавными градиентами температуры рассчитаны плотность потока излучения и источниковый член в задаче для замкнутой полости. Модели, представляющие собой шестипараметрическую дискретно-ординатную модель и две шестипоточные модели типа Шустера–Шварцшильда, оценены с точки зрения точности и экономичности численного счета путем сравнения с известными точными решениями. Сравнение показало, что шестипоточная модель, основанная на делении полости на угловые области точнее и экономичнее двух других моделей.